Exam 9/11/2017

1 A. single electron: $s = 1/2$

nuclear spin: $I = 3/2$

ground state 7s $l=0$, excited state 7p $l=1$

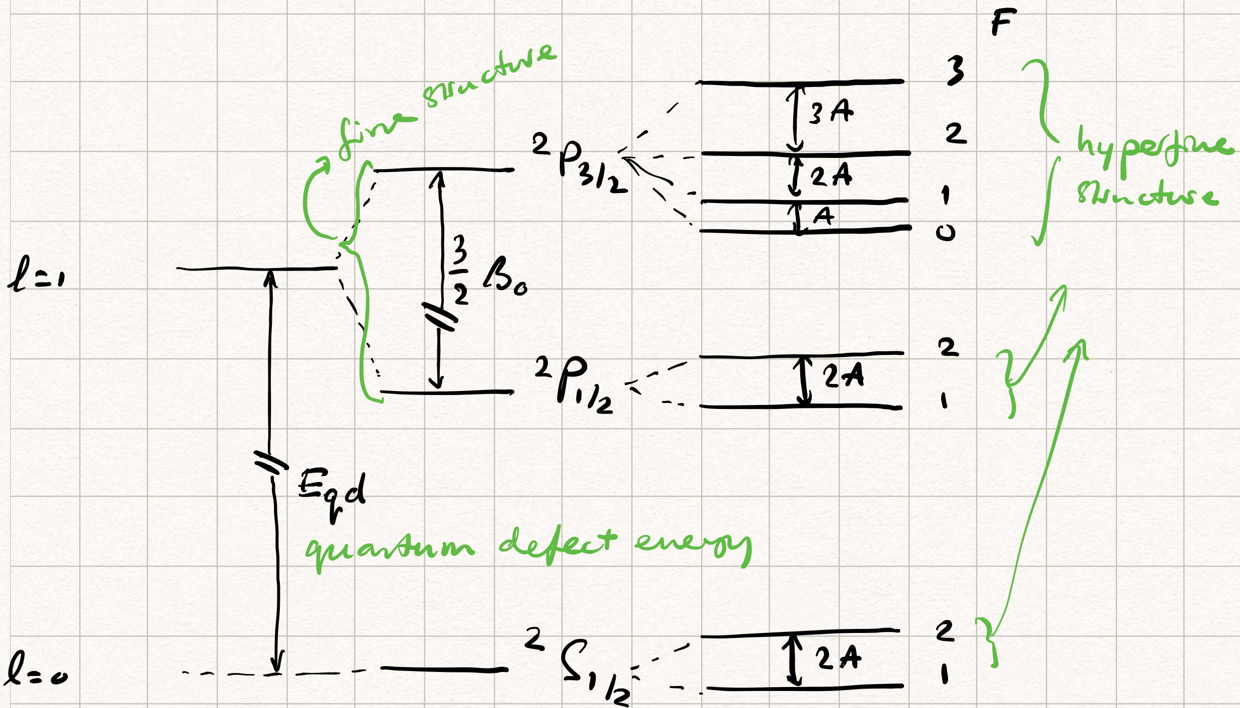
So ground state $^{2S+1}L_J = {}^2S_{1/2}$

excited state ${}^2P_{1/2}$, ${}^2P_{3/2}$

Hyperfine structure in ${}^2S_{1/2}$: $\frac{3}{2}-\frac{1}{2}$, $\frac{3}{2}+\frac{1}{2} \rightarrow F = 1, 2$

${}^2P_{1/2}$: $\frac{3}{2}-\frac{1}{2}$, $\frac{3}{2}+\frac{1}{2} \rightarrow F = 1, 2$

${}^2P_{3/2}$: $\frac{3}{2}-\frac{3}{2}, \ldots \frac{3}{2}+\frac{3}{2} \rightarrow F = 0, 1, 2, 3$



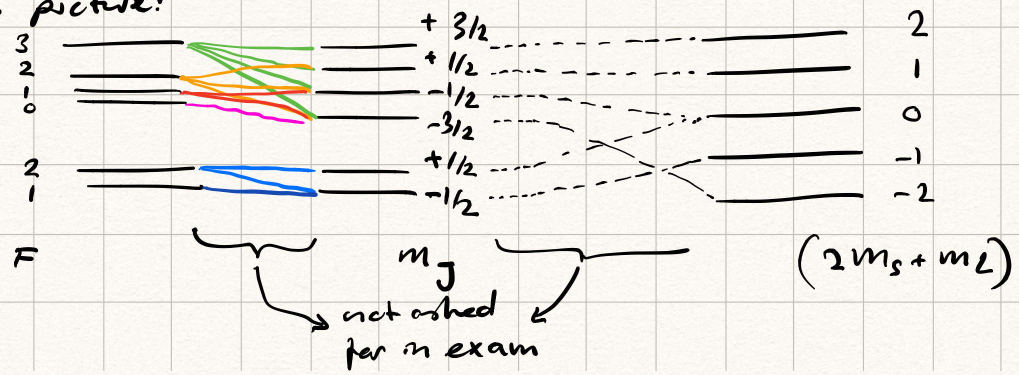
The distance between the energy levels is not drawn to scale, and is determined using the interval rule.

1 B. panel a). At (close to zero) magnetic field, the spacing between the energy levels reflects the hyperfine structure. Splitting due to m_F not yet visible – therefore field zero or very weak. Labels as in 1 A.

panel b) The magnetic field has decoupled $\vec{F} = \vec{I} + \vec{J}$, leading to a four-fold splitting of the $^2P_{3/2}$ ($m_J = +\frac{3}{2}, +\frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}$) and a two-fold splitting of the $^2P_{1/2}$ ($m_J = +\frac{1}{2}, -\frac{1}{2}$). Hyperfine structure is a small perturbation, not visible on this scale.

panel c). Very strong external field, such that $\vec{J} = \vec{L} + \vec{S}$ is decoupled. The energy levels from the $^2P_{3/2}$ and $^2P_{1/2}$ are grouped together, according to $(2m_s + m_\ell) = (2, 1, 0, -1, -2)$.

Total picture:



1 c. To answer this we need to know whether the $m_F = +1$ substate is of low-field seeking or high-field seeking character. So, we have to calculate the g_F factor, since the energy of this level in a weak magnetic field is given by $E = g_F m_F \mu_B B$.

$F = 1$, $l = 1$, $s = 1/2$, $J = 1/2$

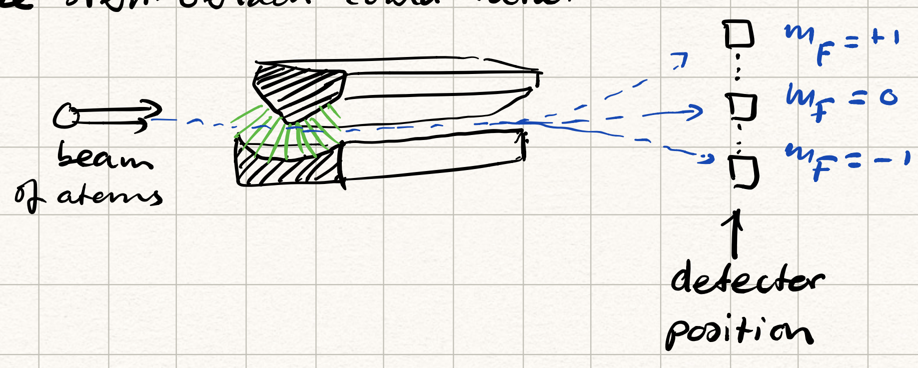
First g_J. it is $1 + \left(\frac{1}{2}\cdot\frac{3}{2} + \frac{1}{2}\cdot\frac{3}{2} - 1\cdot 2\right) / 2\cdot\frac{1}{2}\cdot\frac{3}{2}$

$$= 1 + \left(\frac{3}{4} + \frac{3}{4} - 2\right) / 3/2 = \frac{2}{3}$$

Then $$g_F = g_J \frac{F(F+1) - I(I+1) + J(J+1)}{2F(F+1)} = -\frac{1}{4} g_J = -\frac{1}{6}$$

So $g_F < 0$, which means that the $m_F = +1$ substate is of high-field seeking character - they will move towards a region of <u>high</u> magnetic field.

like Stern-Gerlach could work:



But other solutions might also be correct.

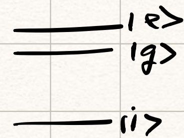
2 A. Large intensity $\rightarrow$ Ω (Rabi frequency) is large. w represents the difference in the population between the two levels : $w = 0$ means equal population.

$$w = \frac{\delta^2 + \Gamma^2/4}{\delta^2 + \Omega^2/2 + \Gamma^2/4} \rightarrow 0 \text{ as } \Omega \text{ becomes large.}$$

Also, as $\Omega = 0$, we see that $w = 1$; then all atoms are in the ground state. When the intensity is increased at most 50% of the atoms is in the excited state.

2 B. Due to movement of the atoms in a room-temperature gas, Doppler broadening results in a Gaussian lineshape, masking the natural (Lorentzian) lineshape. The width of the Doppler broadened absorption profile is of the order of GHz, while the natural linewidth is of the order 1-10 MHz. Since the width of the Doppler profile scales as $1/\sqrt{T}$, the lowering of the temperature by a factor of ~4 only reduces the linewidth by a factor of ~2. To see the natural lineshape, one could use doppler-free spectroscopy or laser cooling.

2 c. three levels:



1) $\Psi = |g\rangle$ initially. Couple $|g\rangle$ and $|i\rangle$ with $\pi/2$ pulse;

2) $\Psi = \frac{1}{2}\sqrt{2}\,|g\rangle - \frac{1}{2}i\sqrt{2}\,|i\rangle$.

now a 2π pulse, coupling $|g\rangle$ and $|e\rangle$;

3) $\Psi = \frac{1}{2}\sqrt{2}\left(\cos\pi\,|g\rangle - i\sin\pi\,|e\rangle\right) - \frac{1}{2}i\sqrt{2}\,|i\rangle$

$= -\frac{1}{2}\sqrt{2}\,|g\rangle - \frac{1}{2}i\sqrt{2}\,|i\rangle$

Now a $\pi/2$ pulse coupling $|g\rangle$ and $|i\rangle$ again:

4) $-\frac{1}{2}\sqrt{2}\left\{\frac{1}{2}\sqrt{2}\,|g\rangle - \frac{1}{2}i\sqrt{2}\,|i\rangle\right\} - \frac{1}{2}i\sqrt{2}\left\{\frac{1}{2}\sqrt{2}\,|i\rangle - \frac{1}{2}i\sqrt{2}\,|g\rangle\right\}$

$= -\frac{1}{2}|g\rangle + \frac{1}{2}i\,|i\rangle - \frac{1}{2}i\,|i\rangle - \frac{1}{2}|g\rangle = -1|g\rangle$

If the 2π pulse would not have been there, the result would be different:

$\frac{1}{2}\sqrt{2}\left\{\frac{1}{2}\sqrt{2}\,|g\rangle - \frac{1}{2}i\sqrt{2}\,|i\rangle\right. - \frac{1}{2}i\sqrt{2}\left\{\frac{1}{2}\sqrt{2}\,|i\rangle - \frac{1}{2}i\sqrt{2}\,|g\rangle\right\}$

$= \frac{1}{2}|g\rangle - \frac{1}{2}i\,|i\rangle - \frac{1}{2}i\,|i\rangle - \frac{1}{2}|g\rangle = -i\,|i\rangle$

So the 2π pulse can be detected by 'sandwiching' between two $\pi/2$ pulses to a third state.

3 A. $$\rho_{22} = \frac{1-w}{2} = \left(1 - \frac{\delta^2 + \Gamma^2/4}{\delta^2 + \Omega^2/2 + \Gamma^2/4}\right)/2$$

$$= \frac{\delta^2 + \Omega^2/2 + \Gamma^2/4 - (\delta^2 + \Gamma^2/4)}{(\delta^2 + \Omega^2/2 + \Gamma^2/4)} \cdot \frac{1}{2}$$

$$= \frac{1}{2} \frac{\Omega^2/2}{\delta^2 + \Omega^2/2 + \Gamma^2/4}$$

$$F_{scatt} = \underbrace{\hbar k}_{\text{momentum of photon}} \cdot \underbrace{\rho_{22}}_{\text{excited state population}} \cdot \underbrace{\Gamma}_{\text{decay rate}}$$

Since $\rho_{22} \to \frac{1}{2}$ as Ω becomes large, the maximum scattering force is

$$F_{max} = \frac{\hbar k \Gamma}{2}.$$

3 B. In a Zeeman slower, atoms are kept in resonance during deceleration by a spatially varying magnetic field. Since the rate of scattering determines the rate of deceleration, a higher scattering rate leads to a shorter Zeeman slower.

The optimal shape follows from the decrease of velocity:

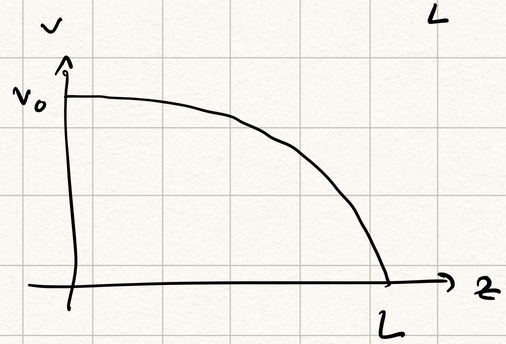
$$\left.\begin{array}{l} v = v_0 - a \cdot t \quad \longrightarrow t = \dfrac{v - v_0}{a} \\ z = v_0 t - \frac{1}{2} a t^2 \end{array}\right\} z = \frac{v_0^2 - v^2}{2a}$$

so that $v^2 = v_0^2 - 2az = v_0^2 - \frac{v_0^2}{L} z = v_0^2 \left(1 - \frac{z}{L}\right)$

Length to stop atoms

$$L = \frac{v_0^2}{2a}$$

so that $v(z) = v_0 \sqrt{1 - \frac{z}{L}}$



The maximum velocity that can still be captured and decelerated: the maximum magnetic field B_0,

$$v_0 \longrightarrow B_0 = \hbar\omega \frac{v_0}{c} \cdot \frac{1}{\mu_B} = \frac{h}{2\pi} \frac{2\pi c}{\lambda} \frac{v_0}{c} \frac{1}{\mu_B} = \frac{h v_0}{\lambda \mu_B}$$

which can be lowered using a bias field and a detuned laser.

3 c. See Foot page 205.

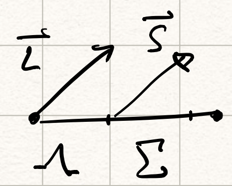
4 A. electronic : X-A @ 663 nm $\rightarrow \frac{c}{\lambda} \approx 4.5 \cdot 10^{14}$ Hz

vibrational : ~ 500 cm^{-1} $\rightarrow$ 15000 GHz $= 1.5 \cdot 10^{13}$ Hz

rotational : ~ 1 cm^{-1} $\rightarrow$ 30 GHz $= 3 \cdot 10^{10}$ Hz

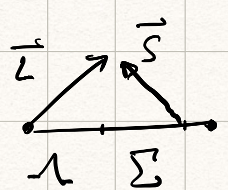
B. $^2\Pi$: $L = 1$ $S = \frac{1}{2}$

$^2\Pi_{3/2}$



$\Omega = \frac{3}{2}$

$^2\Pi_{1/2}$



$\Omega = \frac{1}{2}$

Since $g_S \approx 2$, and $g_L = 1$, the magnetic moment of the 3/2 state is $1 + 1 = 2\,\mu_B$, and of the 1/2 state it is $1 - 1 = 0\,\mu_B$.

C. Dark states are states that the molecules decay into, that they can not be excited from any more. It happens when exciting from a J_{ground} to a $J_{excited}$ state where $J_{ground} > J_{excited}$. This is needed to get a closed rotational transition.

Here an example with π-excitation is given.